SYBA 15/11/17 10 to 1 pages 5

Library

Sem III

(21)

QP Code : 22284

(3 Hours) (Total Marks : 100

N.B : (1) **All** questions are **compulsory**.

(2) Attempt **any two** sub-questions out of three sub-questions from **Q2, Q3, Q4, Q5**.

(3) **Figures** to the right **indicate marks**.

(4) Non-programmable scientific **calculator** is **allowed**.

1. (A) Attempt **any five** from the following : **10**

(i) If odds in favour of 'A' are a : b, then the probability of A^C is ______.

(ii) If $P(A)=\frac{1}{12}$, $P(B)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{52}$ then $P\left(A/B\right)=$ ______ and $P\left(B/A\right)=$ ______.

(iii) If X takes values 0, 1, 2; $P(x=0)=0.6$; $P(x=2)=0.5$ then $P(x=1)$ is -0.1. Is this statement true? Give reason.

(iv) If c is constant var(c) = ______.

(v) State the range of correlation coefficient.

(vi) For a Binomial distribution mean = 5, n = 20, then p = 0.5. Is this true ? Give reason.

(vii) For a Poisson distribution mean = 9 then standard deviation is ______.

(B) Answer the following **(any five)** : **10**

(i) Define Exhaustive events, as applied to the theory of probability.

(ii) State addition theorem of probability for two events, when the events are mutually exclusive.

(iii) If odds in favour of A is 2 : 3, odds against B is 4 : 5. Then $P(A\cap B)=$ ______, if A and B are independent events.

(iv) Define Joint probability mass function of two discrete random variables.

(v) If X and Y are random variables with means 6 and 9 and variances 16 and 25 respectively. If $E(XY)=60$, find $V(X+Y)$.

(vi) State the probability mass function of Binomial distribution.

(vii) Give any two examples of a variable, which follows uniform distribution.

TURN OVER

2. Attempt **any two** sub-questions from the following :

(A) (i) Give classical definition of probability and state its assumptions. **4**

(ii) Three books are selected at random from a shelf consisting of 4 novels, 2 story books and a dictionary. What is the probability that : **6**

(p) 2 novels and 1 story book is selected.

(q) dictionary is not selected.

(r) atleast one story book is selected.

(B) Define with the help of examples the following terms as applied to theory of probability. **10**

(i) Trial.

(ii) Simple event.

(iii) Impossible event.

(iv) Finite sample space.

(v) Mutually exclusive events.

(C) (i) If A and B are two events defined on a sample space 'S' such that P(A), P(B) > 0, Then prove the following : **3**

(p) $P\left(A/_{A^1}\right) = 0$

(q) $P\left(A/_{B}\right) = \frac{P(A)}{P(B)}$ if $A \subset B$.

(ii) Widgets are manufactured in three factories A, B and C. The proportion of defective widgets from each factory are as follows : **7**

Factory A : 0.01

Factory B : 0.04

Factory C : 0.02

Factories A and B produce 30% of the widgets each and the remaining 40% come from factory 'C'.

One widget from the lot is selected and it is found to be a defective. Find the probability that the defective comes from :

(p) Factory A

(q) Factory B.

3. Attempt **any two** sub-questions from the following :

(A) (i) With usual notations show that **6**

$E(aX + b) = aE(X) + b$

$V(aX + b) = a^2\, V(X)$.

(ii) Let X represents the number of weekly credit card purchases a person makes and Y be the number of credit cards a person owns. Suppose a bivariate table for the two variables is as follows : **4**

		X			
		0	1	2	3
Y	1	0.08	0.1	0.1	0.02
	2	0.08	0.05	0.22	0.05
	3	0.04	0.04	0.04	0.18

Evaluate : (p) $P(X \le 1, Y \le 2)$

(q) $P(X \le 2)$

(r) $P(Y = 2)$

(s) $P(X \le 1 / Y \le 2)$

(B) (i) If X and Y are two discrete random variables with joint probability mass function $P(X, Y)$, define marginal distribution of X and conditional distribution of $Y/X = x$. **4**

(ii) The cumulative distribution function of a discrete random variable X is given by : **6**

$$
\begin{aligned}
f(x) &= 0 & x < 0 \\
&= 0.2 & 0 \le x < 2 \\
&= 0.5 & 2 \le x < 4 \\
&= 0.7 & 4 \le x < 6 \\
&= 0.8 & 6 \le x < 8 \\
&= 1 & x \ge 8
\end{aligned}
$$

Obtain expression for p.m.f. (Probability mass function). Hence or otherwise evaluate $P(2 < x \le 6)$; $P(x \ge 4)$; $P(x \le 2)$; $P(0 \le x \le 6)$; $P(x > 6)$.

TURN OVER

(C) (i) Let 'x' be a discrete random variable with probability mass function $P(x)$, then define **4**

(p) r^{th} raw moment.

(q) r^{th} central moment.

(ii) The joint probability mass function if (x, y) is given below. Find $cov(x, y)$. **6**

x \ y	1	2	3
0	0.05	0.10	0.05
1	0.10	0.20	0.15
2	0.10	0.10	0.15

4. Attempt **any two** sub-questions from the following :

(A) Define discrete uniform variate over the range 1, 2, ..., n. Write down its p.m.f. Derive its mean and variance. **10**

(B) (i) State any four applications of Binomial distribution in Real life. **4**

(ii) For a Binomial variate mean is 3, and $15P(x=0)=2P(x=1)$. Find n, p, q, $P(x=5)$, $P(x \geq 2)$, $P(x<2)$. **6**

(C) (i) The calls due to the failure of a computer occur in accordance with Poisson distribution with a mean of 2 per day. Find the probability that : **5**

(p) There are three calls for computer failure on the next day.

(q) Two or more calls on the next day.

(r) Atleast one call on the next day.

[Given $e^{-1}=0.36788$ $e^{-2}=0.13534$].

(ii) State the probability mass function of a Hyper geometric variate and derive its mean. **5**

5. Attempt **any two** sub-questions from the following :

(A) (i) State and prove multiplication theorem of probability for two events. 5

(ii) 5 boys and 2 girls are to be seated in a row. Find the probability that : 5

(p) girls are together.

(q) girls are not together.

(B) (i) Define probability mass function of a random variable 'X' and state its properties. 4

(ii) If X and Y are discrete random variable with joint probability mass function as 6

$$P(x, y) = k(2x + y) \quad x = 2, 3, 4$$
$$y = 2, 3, 4$$
$$= 0 \quad \text{otherwise}$$

Prepare a Bivariate probability distribution table. Also check whether X and Y are independent. Also find marginal probability distribution of 'X'.

(C) (i) Derive the recurrence relation between probabilities for a Binomial distribution. 5

(ii) A taxi cab company has 10 Ambassadors and remaining 5 cars are of other make. A person wants to hire 7 taxis for the marriage party by random choice. Find the probability that he chooses : 5

(p) All ambassadors.

(q) 3 ambassadors.

(r) Also identity the distribution of X, if X is the number of Ambassador cars.
